n-slice algebras of finite type

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1. n-slice algebras and n-hereditary algebras
2. n-slice algebras of finite type

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background

n-hereditary algebra	n-slice algebra
• Iyama'2007. n-Auslander-Reiten translation, n-almost split sequence, maximal $(n-1)$-orthogonal subcategory (n-cluster tilting subcategory)	• Guo'2016. (stable)n-translation quiver(algebra)
• Iyama'2011. absolutely n-complete algebra (n-representation-finite algebra)	• Guo'2020. n-properly-graded quiver(algebra)
• Iyama-Oppermann'2011. n-APR-tilting theory, construct a family of n-representation-finite algebras from a given one	• Guo-Xiao-Lu'2021. n-slice algebra and its classification
• Herschend-Iyama-Oppermann'2014. n-representation-infinite algebra, n-hereditary algebra	• Guo-Lu-Hu-Luo'2023. $\mathbb{Z}\vert_{n-1}Q$

Setting

- k : field
- $\Lambda = \Lambda_0 + \Lambda_1 + \cdots$ is a graded algebra over k with
  - Λ_0 is a direct sum of copies of k,
  - $\Lambda_i \Lambda_j = \Lambda_{i+j}$.

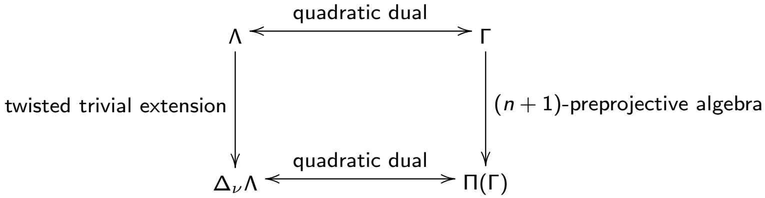
Such algebra is determined by a bound quiver $Q = (Q_0, Q_1, \rho)$ [3].

n-slice algebras

Definition

A quadratic algebra Γ is called an n-slice algebra if its $(n+1)$-preprojective algebra is $(q+1, n+1)$-Koszul for $q \geq 1$ or $q = \infty$ and its quadratic dual is an n-properly-graded algebra.

- *We call $q+1$ the Coxeter index of Γ.*
- *An n-slice algebra is of finite type if q is finite.*



where ν is a graded automorphism of Λ sending an arrow α to $(-1)^n\alpha$.

n-slice algebras and n-hereditary algebras

Theorem

1. *An acyclic n-slice algebra is an n-hereditary algebra.*
2. *Let* Γ *be an acyclic n-hereditary algebra. If its* $(n+1)$*-preprojective algebra* $\Pi(\Gamma)$ *is* $(q+1, n+1)$*-Koszul, then* Γ *is an n-slice algebra.*

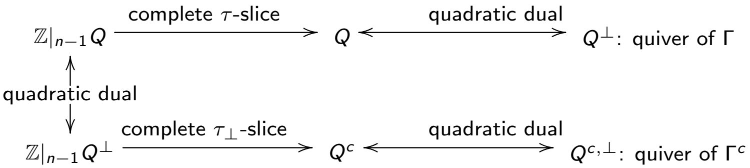
1. *n*-slice algebras and *n*-hereditary algebras
2. *n*-slice algebras of finite type

Higher slice algebras of finite type appear in pairs

Theorem

Assume that Γ is an acyclic n-slice algebra of finite type with Coxeter index $q+1$. Then there is a q-slice algebra Γ^c with Coxeter index $n+1$ such that their repetitive algebras are quadratic dual.

- The q-slice algebra Γ^c is called a companion of Γ.



- $\mathbb{S}(-Q)$: the full subquiver of $\mathbb{Z}|_{n-1}Q$ formed by Q and all the $\tau_{\perp}$-hammocks ending at each vertex of Q.

Theorem

Let Γ be an n-slice algebra of finite type with nicely-graded bound quiver $Q^{\perp}$ and $q+1$ as its Coxeter index and let $\mathcal{Q}$ be the Auslander-Reiten quiver of its n-preprojective modules. Then

1. *There is a q-slice algebra Γ^c of finite type such that $\mathcal{Q}$ is the Auslander-Reiten quiver of the q-preprojective modules of Γ^c.*
2. $\mathcal{Q} = \mathbb{S}(-Q)^{op}$.
3. *As a quiver $\mathcal{Q}$ is the opposite quiver of a quiver obtained by connecting the quiver Q of Γ and the quiver Q^c of Γ^c by some arrows.*

The companion of a path algebra of finite type

Theorem

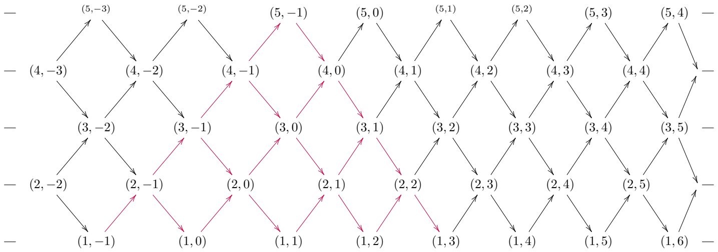
Let Q be the quiver of Dynkin type A, D, E and let Γ be the path algebra of Q. Then the companion Γ^c is of finite representation type.

Example: the path algebra of type A_5

$$Q: \quad 1 \leftarrow 2 \leftarrow 3 \leftarrow 4 \leftarrow 5$$

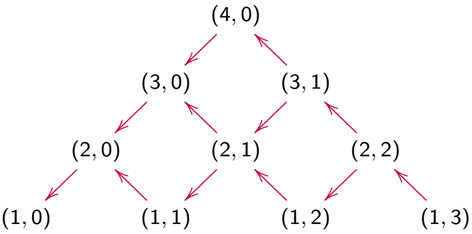
$\Gamma = kQ$ is a 1-slice algebra.

The AR quiver of kQ (the purple part) in $\mathbb{Z}Q^{op}$



Quiver of the companion Γ^c

Q^c :



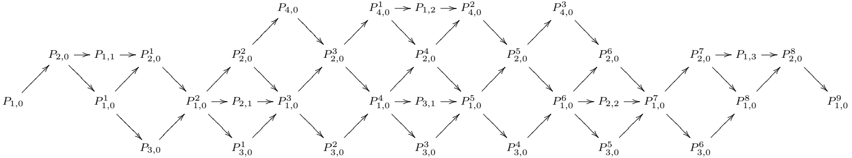
write $\alpha^{\bullet}_{i,t} : (i+1, t) \to (i, t)$, $\beta^{\bullet}_{i,t} : (i-1, t+1) \to (i, t)$,
the relations on Q^c :

$$\begin{array}{ll} \alpha^{\bullet}_{i,t}\beta^{\bullet}_{i+1,t} + \beta^{\bullet}_{i,t}\alpha^{\bullet}_{i-1,t+1} & \text{for } 2 \le i \le 3, 0 \le t \le 3-i \\ \alpha^{\bullet}_{i,t}\alpha^{\bullet}_{i+1,t}, & \text{for } 1 \le i \le 2, 0 \le t \le 2-i \\ \beta^{\bullet}_{i+1,t-1}\beta^{\bullet}_{i,t}, & \text{for } 3 \le i \le 4, 0 \le t \le 4-i \end{array}$$

Γ^c is a 3-slice algebra of finite type.

The AR-quiver of Γ^c

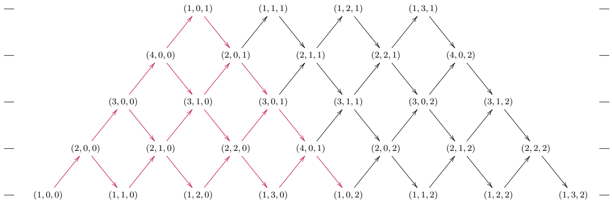
here $P^t_{i,j}$ stands for $\tau^{-t} P_{i,j}$



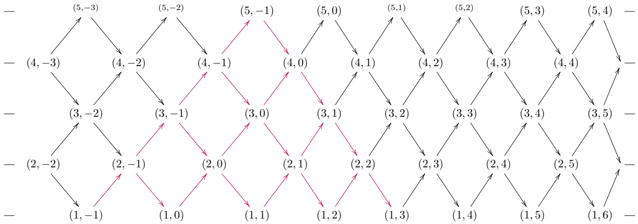
https://github.com/gjy-hn/caARq

The AR-quiver of 3-preprojective modules of Γ^c in $\mathbb{Z}|_2 Q^{c,op}$

from Q^c, take τ-hammocks



from Q, take $\tau_\perp$-hammocks



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Thank you!