Periodicity shadows: a new combinatorial insight in studying periodic algebras

jt work with Jerzy Białkowski and Karin Erdmann

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International Conference on Representations of Algebras (ICRA 21, 2024)

Shanghai Jiao Tong University

Plan

1. Unexpected show up of signed adjacency matrices
2. Periodicity shadows
3. Periodicity shadows of small size

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[Green-Snashal-Solberg, 2003]

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- most of the *weighted surface/triangulation algebras* [Erdmann-Skowroński, 2015]

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where $a_{ij} = a(i,j) - a(j,i)$.

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If Λ is a symmetric algebra with every simple Λ-module periodic of period 4, then its Cartan matrix satisfy the following identity

$$\mathrm{Ad}_Q \cdot C_\Lambda = 0.$$

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Problem 2. Having fixed A such that (Δ) has at least one solution $C \in \mathbb{M}_n(\mathbb{N})$ with non-zero columns, answer when $C = C_\Lambda$ for a tame symmetric algebra Λ with periodic simples (period 4).

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Note: For every algebra $\Lambda = kQ/I$, that is symmetric and all simple Λ-modules are periodic of period 4, the associated adjacency matrix Ad_Q of its Gabriel quiver $Q = Q_\Lambda$ is a periodicity shadow!

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Nevertheless, this already gives a pretty dense sieve. Namely, from the total number of $5^{\frac{n(n-1)}{2}}$ skew-symmetric integer matrices with coefficients in $[-2, 2]$ (and zeros on the diagonal) we have only

- 5 tame periodicity shadows, for $n = 3$,
- 12 tame periodicity shadows, for $n = 4$,
- < 138 tame periodicity shadows, for $n = 5$, and
- < 1290 tame periodicity shadows, if $n = 6$.

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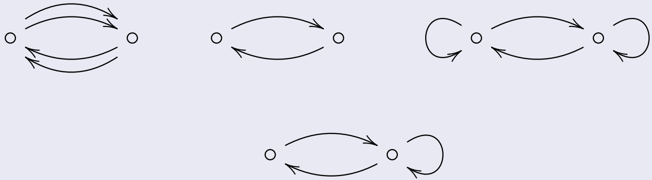

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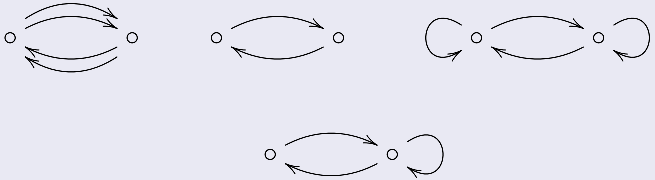

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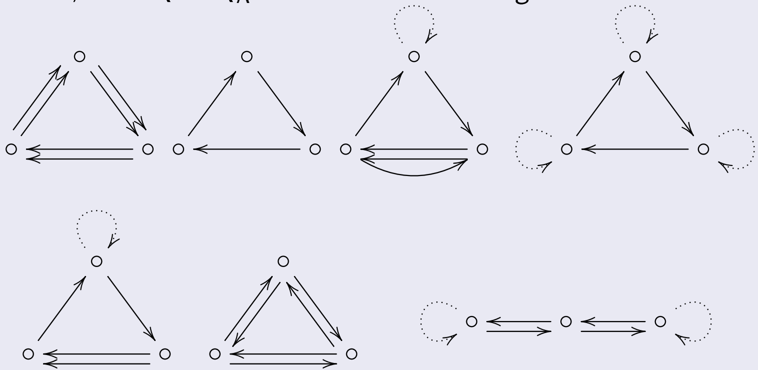

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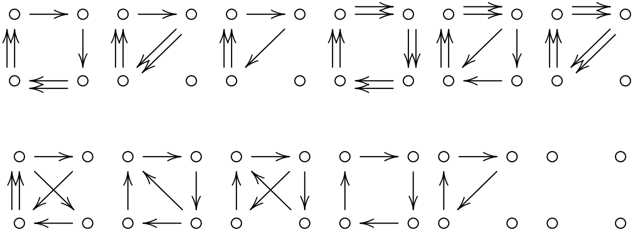
For $n = 4$, we have exactly 12 tame periodicity shadows, identified with the following quivers



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Thank you for your attention !!!