Irreducible representations of the free algebra $K < x_1, \cdots, x_n >$ through Leavitt path algebras

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(joint project with Pham Ngoc Anh)

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The project

The goal: Let K be a field and $x_1, \ldots, x_n$ non commuting variables. We want to investigate the arithmetic of polynomials in $x_1, \ldots, x_n$ (i.e. elements of the free algebra $K < x_1, \cdots, x_n >$), generalising the classical theory for one variable and using Leavitt path algebras. For instance:

- $f \in K[x]$ is irreducible $\Leftrightarrow$ (f) is a maximal ideal $\Leftrightarrow$ $K[x]/(f)$ is a finite-dimensional simple $K[x]$-module. What about if $f \in K < x_1, \cdots, x_n >$? Which are the finite-dimensional simple modules over $K < x_1, \cdots, x_n >$?
- if f and g are in $K[x]$, then the GCD exists and $(d) = (f) + (g)$. What about if $f, g \in K < x_1, \cdots, x_n >$?

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Outline

1. The free algebra $K < x_1, \cdots, x_n >$
2. The Leavitt algebra $L_K(1, n)$
3. Connections
4. Results and open problems

The free algebra $\Lambda = K < x_1, \cdots, x_n >$

1. Λ has a weak division algorithm (i.e. if $\Lambda f \cap \Lambda g \neq 0$, then $f = qg + r$)
2. Λ is a weak Bézout ring (i.e. if $\Lambda f \cap \Lambda g \neq 0$, then $\Lambda f + \Lambda g$ is principal)
3. Λ is a UFR (i.e. for any $f \in \Lambda$, $f = p_1 \cdots p_s$, where the p_i are irreducible and unique up to similarity)

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Definition: The Leavitt algebra $\mathbb{L} = L_K(1, n)$ is the K-algebra with generators $x_1, \cdots, x_n, x_1^* \cdots, x_n^*$ and relations $x_i^* x_i = 1$, $x_i^* x_j = 0$, $x_1 x_1^* + \cdots + x_n x_n^* = 1$.

Remarks:

- $\Lambda \leq \mathbb{L}$ and $\Lambda^* = K < x_1^*, \cdots, x_n^* > \leq \mathbb{L}$.
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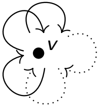

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$\mathbb{L}$ as a perfect localisation of a free algebra

Theorem (Ara-Brustenga '10): The Leavitt algebra $\mathbb{L}$ is a *perfect left localisation* of the free algebra $\Lambda^* = K < x_1^*, \cdots, x_n^* >$

Indeed the canonical inclusion $\Lambda^* \to \mathbb{L}$ is an epimorphism of rings. It is the universal localisation w.r.t a suitable set of maps between fin. gen. projective Λ^*-modules. And $\mathbb{L}$ is flat as right Λ^*-module.

Corollary: The category of finitely presented left $\mathbb{L}$-modules is equivalent to a quotient category of the finite-dimensional left Λ^*-modules (w.r.t a suitable Serre subcategory).

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Corollary: There is a bijection between isomorphism classes of finitely presented simple left $\mathbb{L}$-modules and isomorphism classes of finite dimensional simple left Λ^*-modules.

Remark: The classification problem for finite dimensional simple modules in the free algebra Λ is equivalent to the classification problem in the free algebra Λ^* and so it is equivalent to the classification problem for finitely presented simple modules in $\mathbb{L}$.

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Cofactors

Let $f \in \Lambda$ *comonic* with normal form $f = 1 + \Sigma x_i f_{x_i}$. Define f_{x_i} as the *cofactor* of f of length 1 associated to x_i. Then define recursively cofactors of f of length $m > 1$.

Example: Consider $f = x_1^2 x_2 + x_1 x_2^2 + 1$ in $K < x_1, x_2 >$. The cofactors of f of length 1 are those associated to x_1 and x_2, so $x_1 x_2 + x_2^2$ and 0, respectively. The cofactors of length 2 are those associated to x_1^2 and to $x_1 x_2$, hence x_2. The cofactors of length 3 are those associated to $x_1^2 x_2$ and to $x_1 x_2^2$, hence 1.

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V_f as a left Λ-module

Theorem: Let $f \in \Lambda$ comonic and let V_f as before. Given $\gamma = k + \Sigma x_i \gamma_i \in V_f$, define $x_i \star_f \gamma = -kf_{x_i} + \gamma_{x_i}$, for $i = 1 \ldots n$. Then V_f is a left Λ-module.

Main results

Theorem: Let $f \in \Lambda$ comonic and let V_f as before.

- f is irreducible if and only if V_f is a simple Λ-module
- V_f has finite length.
- If $f = p_1 \cdots p_m$ is a factorization in irreducible polynomials, then m is the length of V_f. The composition factors of V_f are the V_{p_i}'s.
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The role of $\mathbb{L}$ in the proof

Lemma: Let f and g in Λ comonic. Then there exists $d \in \Lambda$ such that $\mathbb{L}f + \mathbb{L}g = \mathbb{L}d$. In particular, if f and g are right coprime, then $\mathbb{L}f + \mathbb{L}g = \mathbb{L}$.

Corollary: If f is irreducible, then $\mathbb{L}f$ is a maximal left ideal and so $\mathbb{L}/\mathbb{L}f$ is a finitely presented simple $\mathbb{L}$-module.

Hence, to conclude the proof, we show that $\mathbb{L}/\mathbb{L}f$ corresponds to V_f in the previously stated bijection, so that V_f is a simple Λ-module.

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Remarks and open problems

From the previous lemma we get that, given f and g in Λ comonic, there exists $d \in \Lambda$ such that $\mathbb{L}f + \mathbb{L}g = \mathbb{L}d$. d is the GCD in Λ and we have an algorithm to find it.

What about f without constant term?

Does the theorem give a complete classification for the finite dimensional simple modules in Λ?

In terms of finite presented simple module over an arbitrary LPA, could we apply this approach to construct new classes of simple modules?

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